

Priority Queue

Processing the shortest job first

⇒ extracting min

Also. insert new printing job into Queue

∴ Priority Queue { Extract-Min
Insert.

⇒ Insert → Priority Queue → Extract-Min

Possible Implementation

1. Unsorted

{ Insert: $O(1)$ (just put it at the end!)
Extract: $O(n)$ (traverse).

2. Sorted

{ Insert: $O(n)$ (ascending order)
Extract: $O(1)$ (first element!)

⇒ Is there any data structure

⇒ Insert, Extract both $O(\log n)$?

(Binary) Heap.

{ Min-heap: Parent node $\leq$ children node
Max-heap

root in array position 1

For any element in position i :

{ left children node: $2i$
right children node: $2i+1$
parent node: $\lfloor i/2 \rfloor$

We can have one-to-one mapping from an array to a heap!

Insertion

① Add new element to the next available position at the lowest level

② "Bubble Up":

if parent of the element $>$ element then: (min-heap)
interchange parent with children (swap)

→ $(\log n)$ levels in total. (n elements)

⇒ running time: $\Theta(\log n)$

Extract-Min:

1. Copy the last element X to the root

⇒ overwrite root!

2. "Bubble down": "zig-zag"

if larger than either of its children:

swap with the smaller one (why smaller one? if large one.

⇒ running time: $\Theta(\log n)$ after swapping "larger" "doesn't satisfy min-heap's rule!"
Smaller

Heap Sort: ① perform n times insert.

② perform n times Extract-Min

⇒ $\Theta(n \log n)$

Example:

