Reasoning Under Uncertainty

Prior Probability (unconditional) : $P(x)$

Posterior Probability (conditional) : $P(y|x)$

Product Rule

$$P(A \wedge B) = P(A)P(B|A) = P(B)P(A|B)$$

$$\Rightarrow \quad P(A|B) = \frac{P(A \wedge B)}{P(A)}$$

Independence

X, Y is independent iff $P(X|Y) = P(X)$

Conditional Independence

X, Y is conditional independent if

$P(X|Y,Z) = P(X|Z)$

Reasoning with Probability

Solution: reasoning with conditional probability

- Use product rule:

$$\begin{aligned} P(B,E,A,J,M) &= P(B,E,A,J)P(M|B,E,A,J) \\ &= \ldots \\ &= P(B)P(E|B)P(A|B,E)P(J|B,E,A)P(M|B,E,A,J). \end{aligned}$$

- Conditional independence:
  - $P(E|B) = P(E)$.
  - $P(J|B,E,A) = P(J|A)$.
  - $P(M|B,E,A,J) = P(M|A)$.
- Thus

$$P(B,E,A,J,M) = P(B)P(E)P(A|B,E)P(J|A)P(M|A).$$

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